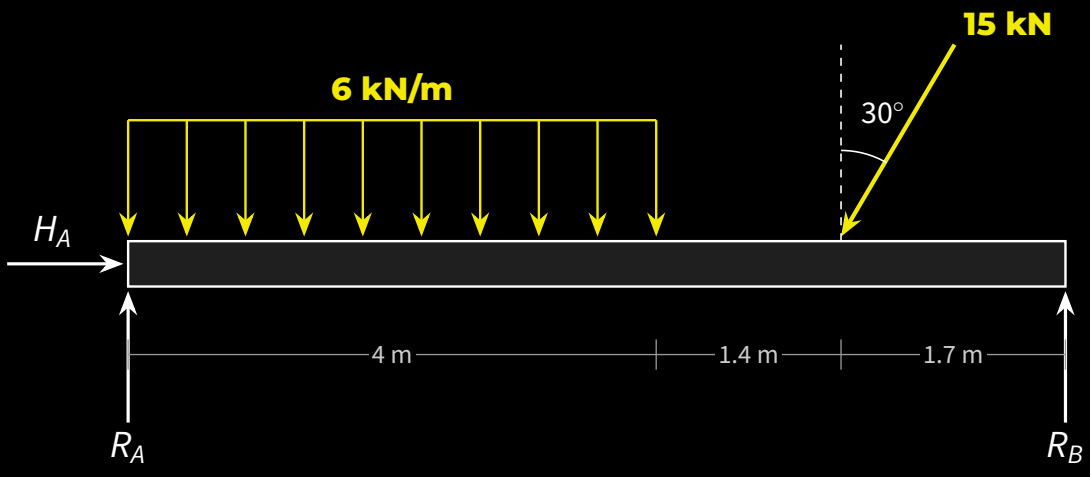
MET ENGINEERING STEP BY STEP • BOOK 1

ENGINEERING MECHANICS

STATICS

Step by Step

Clear concepts, worked examples, practice problems and two full practice exams, all with fully worked solutions



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A. R. BROWN

MET

LOOK INSIDE

Statics Step by Step

This preview shows a selection of pages from the book:

- the contents and how the book works
- the opening of Chapter 1 and all of Section 1.1
- the start of Section 8.3, with a full worked example on a beam with a uniformly distributed load

Some pages are left out, so page numbers jump and links to other parts of the book won't open here.

MET ENGINEERING STEP BY STEP • BOOK 1

ENGINEERING MECHANICS

STATICS

Step by Step

**Clear Concepts, Worked Examples,
Practice Problems and Two Full Practice Exams,
All with Fully Worked Solutions**

Understand why each method works, apply it step by step, and prove you're ready.

WRITTEN BY

A. R. BROWN

MECHANICAL ENGINEERING TUTOR

Step by step, until you can do it yourself.

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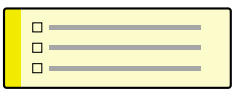
How to use this book

This book takes you from your first force diagram to exam-ready statics, one step at a time, in the same method as the MET video lessons.

It's written for engineering students at college and university, apprentices, and anyone taking a first course in engineering mechanics or learning statics on their own. The maths you need is in *Before you start*, a few pages on.

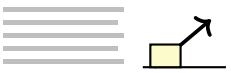
Every section, seven steps

Every section of every chapter is built the same way, so you always know where you are and what comes next.



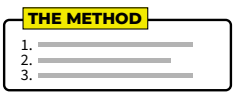
1 What you'll be able to do

Three or four "I can" statements. Read them first so you know where the section is going; tick them off at the end.



2 The idea

The method built from first principles, with a diagram, before any numbers. This is where you find out *why* it works, so you can use it on problems you've never seen.



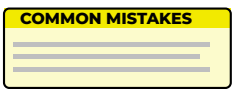
3 The method, step by step

The procedure in numbered steps. The same step names run through the worked examples, the solutions and the videos.



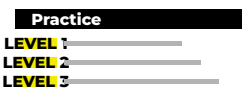
4 Worked examples

Four or five per section, each one testing the idea a different way. The grey notes in the margin say why each step is taken; the yellow bar is the answer.



5 Common mistakes

The traps students fall into, each with a quick check that catches it. Use these checks on every answer you write.



6 Practice in three levels

Level 1 builds one idea at a time, Level 2 combines several steps, Level 3 is exam style. Every problem has a fully worked solution at the back.



7 Watch the lesson

A QR code to the matching MET video. Scan it, or click it in the ebook. Lessons still being filmed say so until they're released.

The four parts of the book

Part	Chapters	What you'll be able to do
A Forces at a point	1 Resolving forces, 2 Resultants, 3 Equilibrium of a particle	split, combine and balance forces that meet at one point
B Moments and rigid bodies	4 Moments and couples, 5 Force system resultants, 6 Equilibrium of rigid bodies	find turning effects and the support reactions of beams, brackets and cranes
C Structures	7 Trusses, frames and machines, 8 Shear force and bending moment diagrams	find the forces inside trusses and beams
D Friction and centroids	9 Friction, 10 Centroids and centre of gravity	decide whether things slip or topple, and find where their weight acts

Each part builds on the one before, so work through the chapters in order. Sections marked *Extension* (the 3D work in Chapters 2, 3 and 4) are for more advanced courses and can be left for later.

Getting the most from the practice

Try first, then check. Work each problem fully on paper before you look. The solutions at the back start with the **boxed final answer**, so you can check quickly, then read the steps only if you need them.

Found a different answer? Read the solution's steps and find the first one where yours went another way. That step is the thing to practise.

Chapter reviews. Each chapter ends with a page of key formulas and the method, an "I can" checklist, and five mixed questions. If you can do those, you're ready for the next chapter.

The ebook. Problem numbers link to their solutions and back, and every "Section" or "Example" in the text is a link.

The two practice exams

When you've finished all ten chapters, sit **Practice exam A** in one go: 2 hours 30 minutes, 100 marks, a calculator and no notes. Mark it with the mark scheme, then use the "where did I lose marks?" grid to see which sections to revise. Revise, then sit **Practice exam B**, which is a little harder. The grade guide after the mark schemes tells you what each score means and what to do next.

Signs and numbers used throughout

Directions	right and up are positive; angles in degrees, always saying what they are measured from
Moments	clockwise positive (Chapters 4 to 10)
Trusses	tension positive: every unknown member force drawn pulling away from its joint
Beams	shear force positive when the force on the left of a cut is upward; sagging bending moment positive
Numbers	SI units; $g = 9.81$ m/s^2; answers to 2 decimal places, with full calculator values carried through the working

A study plan that works

1. Watch the section's video lesson, or read *The idea*.
2. Read the method box, then cover each worked example and try it yourself before reading the solution.
3. Do the Level 1 and Level 2 practice the same day, and Level 3 a day or two later.
4. At the end of each chapter, do the review questions without looking back.
5. Finish with the two practice exams.

CHAPTER

01

Forces and resolving forces

Almost every problem in statics starts the same way: split each force into a horizontal part and a vertical part. Get this step right, with the right size and the right sign, and the rest of the book follows. This chapter builds that skill from the ground up, so you can resolve any force, in any direction, quickly and without sign errors.

IN THIS CHAPTER

1.1 Forces as vectors

1.2 Components of a force

1.3 Sin or cos? Choosing correctly

1.4 Signs in every quadrant

1.5 Forces given by slope or ratio

1.6 Rebuilding a force from its components

Chapter review

Before you start: you need right-angled trigonometry (sin, cos, tan and Pythagoras). If it's rusty, read the toolkit refresher at the front of the book. Set your calculator to **degrees**.

1.1 Forces as vectors

WHAT YOU'LL BE ABLE TO DO

- ☐ I can describe a force fully by its magnitude, direction and line of action.
- ☐ I can tell a scalar from a vector and give examples of each.
- ☐ I can convert between mass and weight using $W = mg$, in N and kN.
- ☐ I can describe the same direction in more than one way, always saying what the angle is measured from.

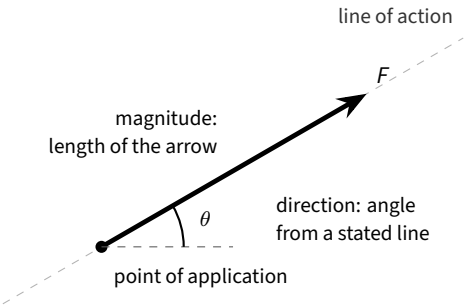
The idea

A force is a push or a pull: a rope pulling a crate, a column pushing up on a beam, the Earth pulling down on everything near it. A force can start something moving, stop it, bend it or hold it still. Statics is about that last job: forces that hold things still.

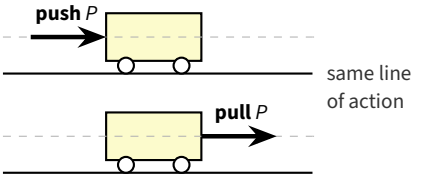
To describe a force completely you need three things:

- its **magnitude**: its size, in newtons (N)
- its **direction**: an angle from a stated line, and which way it points
- its **line of action**: the line it acts along, through its point of application.

So every force in this book is drawn as an **arrow**.



Sliding a force along its own line of action doesn't change its effect on a rigid body. Pushing a trolley from behind or pulling it from the front with the same force, along the same line, does the same job. This is the **principle of transmissibility**. It matters again in Chapter 4, when forces start to turn things.

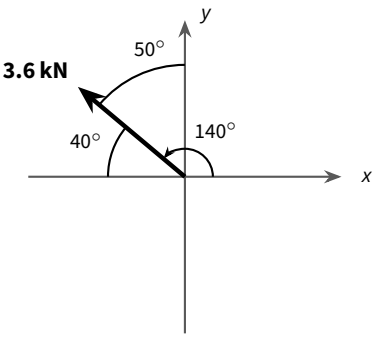


Describing a direction

One direction can be described in several ways. This force acts:

- at 40° above the horizontal, pointing to the left
- at 50° to the vertical, leaning left
- at 140° anticlockwise from the positive x-axis.

All three are correct, as long as you **say which line the angle is measured from**.



Scalars and vectors

A **scalar** needs only a size: 25 kg, 20 °C, 3 s.

A **vector** needs a size **and** a direction: 500 N pulling to the left, 12 m/s due north.

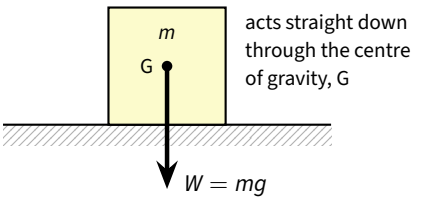
Direction changes how vectors add: two 10 N forces add to 20 N acting the same way, to zero acting opposite ways, and to anything in between at other angles (Chapter 2).

Scalars	Vectors
mass	force
length, distance	weight
time	displacement
temperature	velocity
speed	acceleration
energy	

Mass and weight

Mass is the amount of matter in an object, in kilograms (kg). It is a scalar, and it is the same on Earth, on the Moon or in space.

Weight is the force of gravity on that mass: a vector, in newtons, acting **vertically downwards** through the **centre of gravity**. On Earth, 1 kg weighs 9.81 N:



$W = mg$ with W in N, m in kg and $g = 9.81\ \text{m/s}^2$ $1\ \text{kN} = 1000\ \text{N}$

Sense check: the weight in newtons is roughly ten times the mass in kilograms. A 70 kg person weighs about 700 N; a 1500 kg car about 15 kN.

Describing a force, step by step

1. **Magnitude:** give the size with its unit. Turn a mass into a weight with $W = mg$, and keep N and kN straight.
2. **Direction:** give the angle, the line it is measured from, and which way the force points.
3. **Line of action:** the straight line through the point of application, along the force.
4. **Sketch** the force as an arrow on x and y axes (to scale if asked) and mark the angle.
5. **Check:** N for forces and kg for masses; weight acts straight down; the size makes sense.

Worked examples

These four examples cover the main ways this idea is tested: finding a weight, working back to a mass, comparing mass and weight, and describing one direction in three ways.

WORKED EXAMPLE 1.1.1 The weight of an engine

A mechanic lifts a car engine of mass 180 kg out of a car with an engine hoist. Find the weight of the engine in newtons and in kilonewtons, and describe the weight fully as a vector.

Step 1: Magnitude.
$W = mg = 180 \times 9.81 = 1765.80$ N
$= 1765.80 \div 1000 = 1.77$ kN
Mass in kg times g gives newtons. Divide by 1000 for kilonewtons.

Step 2: Direction. Vertically downwards.
Weight always acts straight down, whatever the object hangs from or rests on.

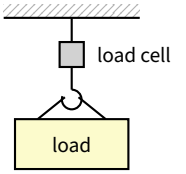
Step 3: Line of action. A vertical line through the engine's centre of gravity.
Chapter 10 shows how to find the centre of gravity.

Step 5: Check. 1765.80 N is roughly ten times 180. ✓
The "ten times" check catches a forgotten g straight away.

Answer: $W =$ **1765.80** N $=$ **1.77** kN, acting vertically downwards through the centre of gravity.

WORKED EXAMPLE 1.1.2 From a load-cell reading to a mass

A load cell on a crane hook measures the force in the cable. It reads 24.5 kN while the load hangs still. What mass is hanging from the hook?



Step 1: Magnitude. The load hangs still, so the cable force equals the weight: $W = 24.5\ \text{kN} = 24500\ \text{N}$.

Convert kN to N before using g, because g is in m/s².

Rearrange.

$$W = mg \ \Rightarrow\ m = \frac{W}{g} = \frac{24500}{9.81} = 2497.45\ \text{kg}$$

Working back: divide by g to go from newtons to kilograms.

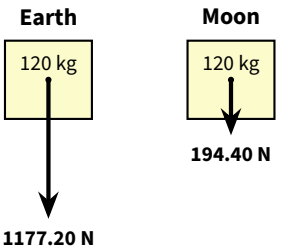
Step 5: Check. $2497.45 \times 9.81 = 24499.98$ N, which is 24.5 kN to rounding. ✓ About 2.5 tonnes: a sensible crane load.

Put your answer back into the formula you started from.

Answer: $m =$ **2497.45** kg (about 2.5 tonnes).

WORKED EXAMPLE 1.1.3 Same mass, different weight

A spacesuit and its life-support pack have a mass of 120 kg. Find their weight on Earth and on the Moon, where $g = 1.62\,\text{m/s}^2$. Which of the two quantities, mass or weight, is the same in both places, and why?



Step 1: Magnitude.

Same formula, different g.

Earth: $W = 120 \times 9.81 = 1177.20$ N

Moon: $W = 120 \times 1.62 = 194.40$ N

Step 2: Direction. Both act vertically downwards, towards the centre of the body the suit stands on.

"Down" means towards the Earth or the Moon.

Compare. $\dfrac{1177.20}{194.40} = 6.06$, so the suit weighs about six times more on Earth.

The ratio is just $9.81 \div 1.62$.

Step 5: Check. The mass, 120 kg, is the same in both places: the amount of matter doesn't change. Weight depends on gravity and has a direction.

Mass is a scalar; weight is a force, so it is a vector.

Answer: Earth: **1177.20** N; Moon: **194.40** N. The **mass** stays the same; the weight changes with g.

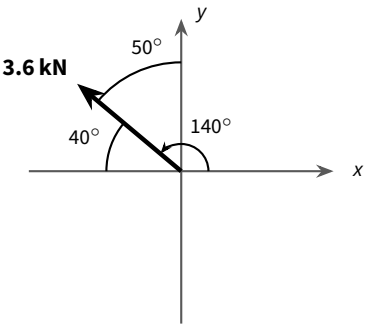
WORKED EXAMPLE 1.1.4 One force, three descriptions

A cable pulls on a ring with a force of 3.6 kN, at 40° above the horizontal, pointing up and to the left.

(a) Describe its direction as an angle from the vertical.

(b) Describe it as an angle measured anticlockwise from the positive x-axis.

(c) How long is the arrow at a scale of 1 cm to 1 kN?



Step 4: Sketch. Up and to the left: the arrow lies between the positive y-axis and the negative x-axis.

Sketch first, so every angle has a picture to check against.

(a) Direction from the vertical. The horizontal and vertical are 90° apart: $90^\circ - 40^\circ = 50^\circ$ to the vertical, leaning left.

Angle from horizontal + angle from vertical = 90°.

(b) Direction from $+x$. Turning anticlockwise from $+x$ to the $-x$ axis is 180°; the force is 40° short of that: $180^\circ - 40^\circ = 140^\circ$.

This form is used in Section 1.4 to get the signs automatically.

(c) Magnitude to scale. 3.6 kN at 1 cm per kN gives an arrow 3.6 cm long.

Pick a scale that makes the drawing a sensible size.

Step 5: Check. 140° is between 90° and 180°, which is up and to the left. ✓

Each description points the same way.

Answer: (a) **50**° to the vertical, leaning left (b) **140**° anticlockwise from $+x$ (c) **3.6** cm

COMMON MISTAKES

- **Giving a weight in kg, or a mass in N.** *Quick check:* masses are in kg, every force (weight included) is in N or kN.
- **Losing a factor of 1000 between N and kN.** *Quick check:* a person weighs about 0.7 kN and a car about 15 kN. Does your answer fit?
- **Giving a force without a direction.** A force of 500 N is only half an answer. *Quick check:* does your answer say which way it acts?
- **Giving an angle without its reference line.** "30°" could be from the horizontal, the vertical or the x-axis. *Quick check:* say "from the horizontal", "to the vertical" or "anticlockwise from $+x$".

Practice 1.1

Full worked solutions start on page 464. Work on paper and sketch every force first.

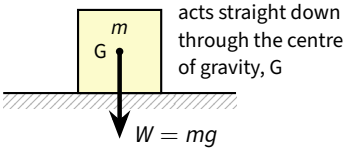
LEVEL 1 Build: one idea at a time

1.1.1 Find the weight of each object. Give (b) in kN.

(a) a 75 kg person

(b) a 1450 kg car

(c) a 350 g phone



1.1.2 Say whether each quantity is a scalar or a vector, and give a reason.

(a) the mass of a bag of cement, 25 kg

(b) the weight of the same bag

(c) a velocity of 12 m/s due north

(d) a temperature of 20 °C

(e) a pull of 5 kN along a cable at 30° to the horizontal

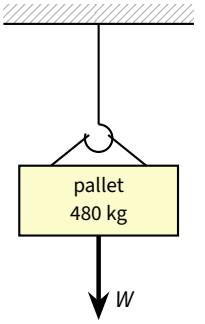
(f) a speed of 12 m/s

LEVEL 2 Apply: several steps combined

1.1.3 A lifting hook has a safe working load of 5 kN: the largest force it may carry.

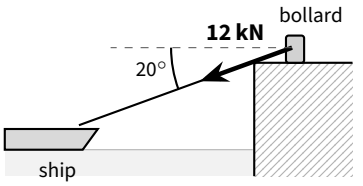
(a) What is the largest mass the hook may lift?

(b) A pallet of mass 480 kg is wrapped in 35 kg of packing. May the hook lift it? By how many newtons is it over or under the limit?



LEVEL 3 Exam style: real contexts and multi-part questions

1.1.4 A mooring rope runs down from a bollard on a quay to a ship. The rope pulls on the bollard with a force of 12 kN, at 20° below the horizontal, pointing down and to the left.



(a) Describe the direction of the force as an angle from the vertical.

(b) Give the direction as an angle measured anti-clockwise from the positive x-axis.

(c) How long is the arrow at a scale of 1 cm to 2 kN?

(d) What mass would have a weight equal to 12 kN?

(e) The ship drifts further out, but the rope stays on the same line with the same tension. Does the force on the bollard change? Explain.

WATCH THE LESSON

Forces as vectors: video coming soon

This lesson is being filmed. Until it's ready, the code opens a page that says so and links to the lesson library; when the video is released, the same code opens it.

Scan the code or go to `mechanicalengineeringtutor.com/v/1-1`

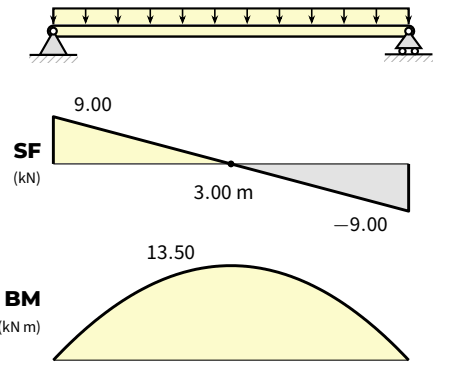
8.3 Uniformly distributed loads

WHAT YOU'LL BE ABLE TO DO

- ☐ I can draw the SFD and BMD of a beam carrying a uniformly distributed load (UDL).
- ☐ I can find where the shear force is zero under a UDL, and the maximum bending moment there.
- ☐ I can draw the diagrams for UDLs and point loads together, and for a UDL on part of the span.
- ☐ I can use $M_{\max} = wL^2/8$, and work back from a bending moment limit to the largest UDL.

The idea

Under a UDL the shear force doesn't jump: it falls steadily, by w for every metre you walk, so the SFD is a **sloping straight line**.
The bending moment still grows by the shear force times the distance, but the shear force keeps shrinking, so the BMD bends over into a **curve** (a parabola). The curve peaks, flat on top, exactly where the shear force has fallen to **zero**; after that the shear force is negative and the bending moment falls.



Under a UDL	SF: a sloping line, falling w per metre; BM: a curve
Zero shear	$x = V/w$, measured from where the slope starts, V being the SF there
Full-span UDL	$R_A = R_B = wL/2$; $M_{\max} = wL^2/8$ at mid-span

For a full-span UDL, at mid-span the left half carries $R_A = wL/2$ at $L/2$ and its own load $wL/2$ at $L/4$, so $M = \frac{wL}{2} \cdot \frac{L}{2} - \frac{wL}{2} \cdot \frac{L}{4} = \frac{wL^2}{8}$. It only works for that one case: with any other loading, find where the shear force is zero.

The SFD and BMD with UDLs, step by step

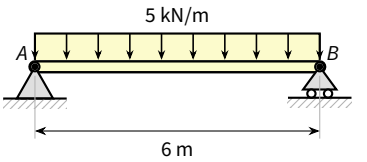
1. **Reactions:** replace each UDL by $w\times$ its length at its middle, then take moments.
2. **Shear force:** find it at each key point: supports, point loads, and where each UDL starts and stops. Join with sloping lines under UDLs, flat lines elsewhere.
3. **Zero shear:** where a sloping part crosses the axis, $x = V/w$ from the start of that slope.
4. **Bending moment:** at each key point and at zero shear, from the moments on one side (a UDL part counts as $w\times$ its length, at its middle).
5. **Draw and check:** curves under UDLs, straight lines elsewhere; the curve peaks where the SF is zero.

Worked examples

These five examples cover the main ways this idea is tested: a full-span UDL, a UDL with a point load (twice, with the maximum in different places), a UDL on part of the span, and working back to the largest UDL.

WORKED EXAMPLE 8.3.1 A full-span UDL

A simply supported beam AB spans 6 m and carries 5 kN/m over its whole length. Draw the SFD and BMD.



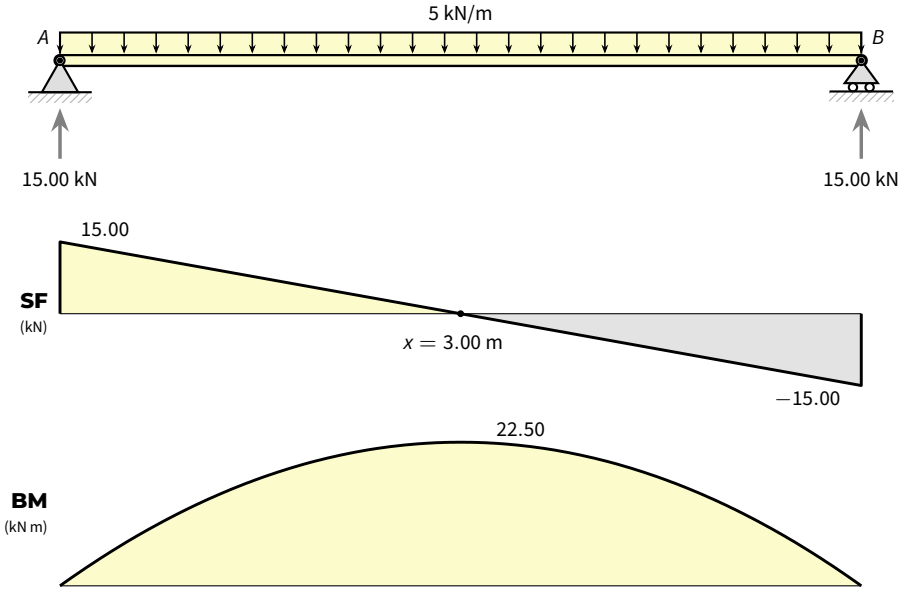
Step 1: Reactions. Total load $5 \times 6 = 30$ kN, central: $R_A = R_B = 15.00$ kN — *Symmetric.*

Step 2: Shear force. 15.00 at A, falling 5 kN per metre to $15.00 - 30 = -15.00$ at B — *A straight sloping line.*

Step 3: Zero shear. $x = 15.00/5 = 3.00$ m: mid-span — *$x = V/w$.*

Step 4: Bending moment. At 3 m: $M = 15.00 \times 3 - 15 \times 1.5 = 45.00 - 22.50 = 22.50$ kN m — *The left half of the UDL: 15 kN at 1.5 m from the cut.*

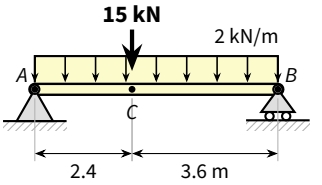
Step 5: Check. $wL^2/8 = 5 \times 36/8 = 22.50$ kN m. ✓



Answer: SF 15.00 to −15.00 kN; $M_{max} =$ **22.50** kN m at mid-span

WORKED EXAMPLE 8.3.2 A UDL and a point load

A simply supported beam AB spans 6 m. It carries 2 kN/m over its whole length and 15 kN at C, 2.4 m from A. Draw the SFD and BMD, and find the maximum bending moment.



Step 1: Reactions. UDL 12 kN at 3 m. About A:
$R_B \times 6 = 12 \times 3 + 15 \times 2.4 = 72$, $R_B = 12.00$;
$R_A = 27 - 12.00 = 15.00$ kN

Step 2: Shear force. 15.00 at A; just left of C:
$15.00 - 2 \times 2.4 = 10.20$; just right: $10.20 - 15 = -4.80$;
at B: $-4.80 - 2 \times 3.6 = -12.00$ kN

Sloping under the UDL, with a step at the load.

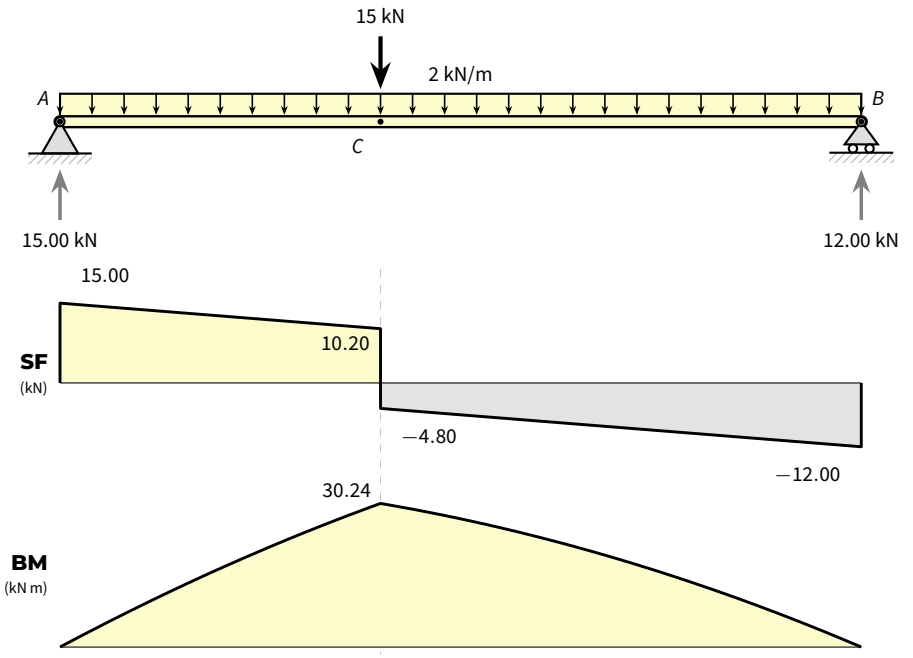
Step 3: Zero shear. The SF never crosses zero on a slope: it jumps from $+10.20$ to -4.80 at C. The maximum is at C.

Changes sign at a point load.

Step 4: Bending moment.
$M_C = 15.00 \times 2.4 - 4.8 \times 1.2 = 36.00 - 5.76 = 30.24$ kN m

2.4 m of UDL: 4.8 kN at 1.2 m.

Step 5: Check. From the right:
$12.00 \times 3.6 - 7.2 \times 1.8 = 43.20 - 12.96 = 30.24$. ✓



Answer: $M_{max} =$ **30.24** kN m at C, under the point load

That was a look inside.

The full book has 10 chapters, more than 230 worked examples, practice questions at three levels and two full practice exams, all with fully worked solutions.

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